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What is maths mastery?

Essentially, maths mastery promotes high expectations for all learners. It aims to give all pupils an understanding of the structures and concepts behind mathematics to encourage children to question how and why different methods are used in order to develop deep mathematical understanding. The guiding principles of teaching for mastery, identified by the National Centre of Excellence in the Teaching of Mathematics (NCETM), are:

- Coherence – the small steps to reveal mathematical concepts.
- Representation and structure – concrete, pictorial and abstract representations to access mathematical structures.
- Mathematical thinking – developing reasoning skills and making connections.
- Fluency – number facts and multiplication ~~times~~ tables facts.
- Variation – procedural and conceptual variation of mathematical concepts.

The resources in this book explore these principles and expose the underlying structures of mathematics, to allow children to reason fluently and make connections. Children need to be explicitly taught to reason mathematically. When mathematics becomes solely about getting the right answer it removes the focus on concepts. When an answer is given, children need to be encouraged to consider **how** and **why** it is the correct answer.

Fluency is more than being 'quick at mathematics'. It requires a deep understanding of facts and the ability to connect this understanding to problems.

Planning for mastery

Planning for mastery requires careful consideration of the concepts you wish to expose. Key vocabulary ought to be introduced early on and then problems should be designed to use the key vocabulary, allowing children to easily access the problems using a common mathematical language to explain their thinking.

Build on concepts using small steps. For example, start with addition using place value counters, then move on to addition using carrying, before moving to addition in the abstract. Use all these methods in tandem and alongside each other.

Giving examples of new concepts alongside non-examples can be a powerful way to expose children to new concepts. For example:

Concept: a triangle		
What it is 	What it is 	What it is not 

Any independent tasks should reflect the structure of the tasks that you have modelled to children.

You can find the following online at **www.mrbeeteach.com/**:

- Examples of concept maps, to explore the idea of what a concept is and is not.
- A handy lesson starter grid (and an example to show how it can be used). The grid invites children to focus on what they learnt yesterday, last week, last term and last year.

Concrete, pictorial, abstract

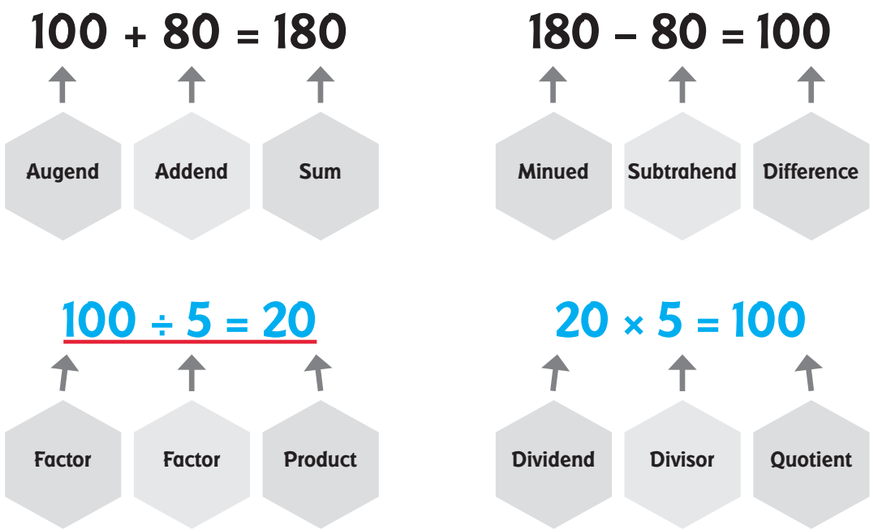
The images in this book have been chosen to support a concrete, pictorial, abstract (CPA) approach to mathematics. Many of the activities can be reimagined as practical/kinaesthetic resources whereby the children can use manipulatives such as counters, Cuisenaire rods, base 10 sets and place value counters to reveal mathematical structures and concepts. The CPA approach can allow children to better understand and reason about the mathematics they encounter.

Vocabulary

Mathematical reasoning requires specific mathematical vocabulary, without which precision and clarity can be lost. The activities in the book are supported with key vocabulary linked to each concept. This vocabulary needs to be explicitly taught and understood in order for children to be able to communicate and reason clearly.

Below is some key vocabulary for the four operations: addition, subtraction, multiplication and division.

Augend	The number to be added to in an addition.
Addend	The number added to the augend.
Sum	The total of the augend and addend.
Minuend	The first number in a subtraction. A number from which an amount will be taken.
Subtrahend	The number to subtract from the minuend.
Difference	The result of subtracting the subtrahend from the minuend.
Multiplier (factor)	The number that gets multiplied.
Multiplicand (factor)	The numbers of times to multiple.
Product	The answer when two numbers are multiplied together.
Dividend	The amount to be divided.
Divisor	The number to divide by.
Quotient	The answer when we divide the dividend by the divisor.



Part, whole

The key vocabulary of 'part, whole' appears throughout the book. It's an essential concept for children to understand as it links to many areas of the maths curriculum, such as place value, the four operations, fractions, ratio, measurement and statistics. Using the words 'part' and 'whole' can allow children to break down a concept and reveal the structure of a problem.

For example, children might say the following about the number 346:

The whole is 346.
6 (six) is a part.
40 (forty) is a part.
300 (three hundred) is a part.

It is essential that children listen to and say the key vocabulary. A 'stem sentence' approach can be useful for teaching and using mathematical vocabulary.

Stem sentences

The activities in the book come with stem sentences, which allow teachers to support the children's thinking when reasoning and give structure to their answers. They also allow children to focus on key vocabulary.

Teaching for mastery involves having high expectations for all learners, who are exposed to the same mathematical structures using precise and technical vocabulary.

The expectation is that children will speak in full sentences. Rather than this:

Teacher: 'What is the product of 6 and 5?'
Child: '30.'

Try this:
Teacher: 'What is the product of 6 and 5?'
Child: 'The product of 6 and 5 is 30.'

Stem sentences work best when using the 'I say, you say, we say' approach.

I say, you say, we say

I say	The teacher reads the complete stem sentence to the class.
You say	The teacher picks a selection of children to read the stem sentence out loud to the class.
We say	The whole class says the stem sentence together.

If… then…

This book includes many examples of stem sentences using the structure of 'If… then…'. It allows children to make connections and look for patterns. For example:

Child: 'If 3 x 10 = 30, then 30 x 10 = 300.'

Once embedded, this approach can change how children think about mathematics.

Variation theory

Essentially, there are two ways variation can support mathematical teaching and learning: procedural and conceptual variation.

Procedural variation is about exposing patterns and connections. For example, looking for number bonds to 5 (0 + 5, 1 + 4, 2 + 3, etc.) and apply those to numbers one hundred times greater (0 + 500, 100 + 400, 200 + 300, etc.).

Conceptual variation is about showing different representations of the same task or concept. For example, showing what 10 can look like in many different ways:

- 10 + 0, 1 x 10 or 20 – 10
- part-whole models
- bar models
- place value counters
- cubes and other manipulatives.

Asking 'What's the same? What's different' can be a powerful strategy to encourage children to focus on similarities and differences between mathematical concepts.

Addition and subtraction 2

These activities expose the structure of addition. Children add ones to a two-digit number, add two two-digit numbers crossing the tens barrier, or add three one-digit numbers.

Curriculum notes

By the end of KS1, children should be able to:

- add and subtract numbers using concrete objects, pictorial representations, and mentally, including a two-digit number and ones, a two-digit number and tens, two two-digit numbers or three one-digit numbers.

Teacher notes

1. Introduce the 'Before you start' activity. You could use a number line or another counting resource to show how to find the sum of each calculation. The 'Before you start' activities on these two pages are designed to show students how the same sums can be made with different number combinations.
2. Model the whole class activity. Show a few different solutions such as 20 + 1 = 21, or 10 + 11 = 21.
3. Draw attention to the way children partition the numbers.
4. For activity 2, note the variation in the positioning of the equals ~~symbol~~ and the positioning of the sum/answer.

Stem sentences for the whole class activity

20 + 1 = 21.

10 + 10 + 1 = 21.

WHOLE CLASS ACTIVITY

Before you start: 10 + 10 = ___ 7 + 7 + 7 =___ 11 + 11 = ___

Make the statements correct.

$\square + \square = 21$

$\square + \square + \square = 21$

Stem sentences to support reasoning

___ + ___ = 21.

___ + ___ + ___ = 21.

Addition and subtraction 2

Vocabulary: part, whole, ones, tens, place, value, partition

ACTIVITY 1

Before you start: 50 + 40 = ___ 50 + 20 + 21 = ___ 50 + 41 = ___

Make the statements correct.

$\square + \square = 22$

$\square + \square + \square = 22$

Stem sentences to support reasoning

___+ ___= 22.

___+ ___+ ___= 22.

ACTIVITY 2

Before you start: 60 + 40 = ___ 60 + 20 + 21 = ___ 60 + 41 = ___

~~What is the sum of each row and column?~~

Make the statements correct.

multiple of 10

$80 = \square + \square$

$\square + \square + \square = 80$

Stem sentences to support reasoning

80 = ___+ ___

80 = ___+ ___+ ___

or

___+ ___= 80.

___+ ___+ ___= 80.

Addition and subtraction 3

These activities focus on addition in the context of hair growing.

Curriculum notes

By the end of KS1, children should be able to:

- solve problems with addition and subtraction, using concrete objects and pictorial representations, including those involving numbers, quantities and measures.

Teacher notes

Children could use number lines or other pictorial strategies used in school.

1. Introduce the 'Before you start' activity. Draw attention to the addend increasing by 10 each time and noticing the effect on the answer by looking for patterns.
2. Model the whole class activity. Explain that we know Lynsey's hair is 25cm long and grows 5cm more.
3. Show the structure of this problem as 25 + 5 = 30 or represent it using a bar model.

Stem sentences for the whole class activity

25cm + 5cm = 30cm.

WHOLE CLASS ACTIVITY

Before you start: 13cm + 10cm = ___ ___= 13cm + 20cm 13cm + 30cm = ___

In February, Lynsey's hair measures 25cm.

Lynsey's hair grows 5cm.

How long is her hair now?

Stem sentences to support reasoning

___cm + ___cm = ___cm.

Addition and subtraction 3

Vocabulary part, whole, ones, tens, addition, more

ACTIVITY 1

Before you start: 33cm + 10cm = ___ ___= 43cm + 20cm 53cm + 30cm = ___

In May, Lynsey's hair measures 35cm.

Lynsey's hair grows 15cm.

How long is her hair now?

Stem sentences to support reasoning

___cm + ___cm = ___cm.

ACTIVITY 2

Before you start: 33cm + 15cm = ___ ___= 43cm + 15cm 53cm + 15cm = ___

The table shows the length of Lynsey's hair on different months of the year.

How much did Lynsey's hair grow between June and August?

May	45cm
June	60cm
July	71cm
August	87cm

Stem sentences to support reasoning

___cm – ___cm = ___cm.

Addition and subtraction 4

These activities focus on concepts surrounding addition, with limitations on the types of answers accepted.

Curriculum notes

By the end of KS1, children should be able to:

- recall and use addition and subtraction facts to 20 fluently and derive and use related facts up to 100.

Teacher notes

1. Model the 'Before you start' activity. Use place value counters, 100 squares or other manipulatives. Draw attention to the pattern and what happens when adding 10 to a number.
2. Model the whole class activity. Explain that the answer must be in the 5 times table. There are many possibilities and the sum could be represented as a part-whole model or a bar model.
3. Show some examples and non-examples which work and do not work. Using counters can help children see the calculations.
4. For activities 1 and 2, explain that children can use the same card twice. So 4 + 4 = 8 would be a possible stem sentence for activity 1.

Stem sentences for the whole class activity

6 + 4 = 10 (accept 4 + 6).
8 + 2 = 10.
12 + 8 = 10.
14 + 6 = 10.
16 + 4 = 10.

WHOLE CLASS ACTIVITY

Before you start: 19 + 10 = ___ ___= 29 + 10 ___ = 39 + 10

Using the digit cards, how many ways can you make a correct sum?

2	4	6	8	10	12	14	16

□ + □ = □

multiple of 5

Stem sentences to support reasoning

___+ ___= ___.

Addition and subtraction 4

Vocabulary: part, whole, ones, tens, addition, multiple

ACTIVITY 1

Before you start: 29 + 20 = ___ ___ = 39 + 20 ___= 49 + 20

Using the digit cards, how many ways can you make a correct sum?

2	4	6	8	12	14	16	18

□ + □ = □

multiple of 4

Stem sentences to support reasoning

___+ ___= ___.

ACTIVITY 2

Before you start: 29 + 10 = ___ ___= 49 + 10 ___= 69 + 10

Using the digit cards, how many ways can you make a correct sum?

3	6	9	12	15	18	21	24

□ + □ + □ = □

multiple of 3

Stem sentences to support reasoning

___+ ___+ ___= ___.